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Use of Padé Fractions in the Calculation of Blunt Body Flows

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IN Ref. 1, a method involving Padé fractions is given for calculation of the axially symmetric flow of a perfect gas past a blunt body of revolution. Terms of the Taylor expansion of the stream function in the neighborhood of the nose of the shock up to and including degree 8 are used, and are calculated by means of explicit expressions due to Lin and Shen.[2] This Taylor expansion does not converge at the body,[3] and the use of Padé fractions has been found to be a means of obtaining convergence.[4] A related method, using continued fractions, has been given by Moran.[5] Other methods for removing the divergence of series expansions in the blunt body problem are given in Refs. 6–8.

In the present Note, the method of Ref. 1 is generalized to include calculation of an arbitrary number of terms of the Taylor expansion of the stream function. Calculations are carried out in several cases using the present method together with a method of characteristics program written for the IBM 7090 by Thompson and Furey.[9]

A uniform flow of a perfect gas is assumed ahead of the bow shock, with Mach number M_∞ and ratio of specific heats γ. Let x and r be cylindrical coordinates with origin at the nose of the body, and let the shock be given by

$$(r/R_s)^2 = 2(x + x_0)/R_s - B_s(x + x_0)^2/R_s^2 + \dots \tag{1}$$

where R_s is the radius of curvature of the shock nose, B_s is the shock bluntness, and x_0 is the shock detachment distance. The equation of the body which would produce the shock is

$$(r/R_b)^2 = 2(x/R_b) - B_b(x/R_b)^2 + \dots \tag{2}$$

where R_b is the radius of curvature of the nose, and B_b is the bluntness.

In the inverse problem, in which the shock is given and the body which would produce it is calculated, we first find coefficients of the Taylor expansion of the stream function in cylindrical coordinates. Bernoulli's equation and the vorticity equation are used as the starting point of this calculation, as in the procedure of Ref. 2. Subroutines for power series manipulations are used, based in part on algorithms due to Leavitt.[10] As in Ref. 1, a transformation is then made to coordinates τ and ξ such that the shock is the coordinate surface $\tau = 0$. The Taylor expansion of the stream function in the new coordinates will be written in the form

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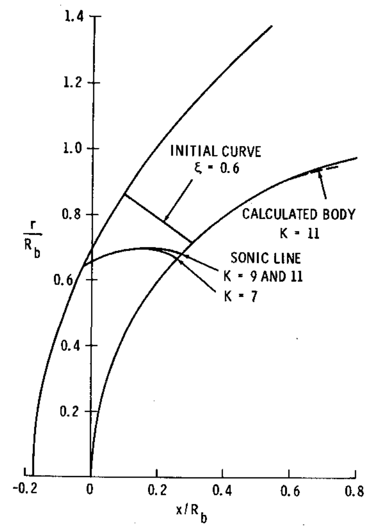

Fig. 1 Sonic line and initial curve in the flow past a sphere at $M_\infty = 4$.

$$\frac{\psi}{\rho_\infty q_\infty R_s^2} = \sum_{i=1}^{\infty} \psi_i(\tau)\xi^{2i} \tag{3}$$

where ρ_∞ and q_∞ are the density and velocity magnitude, respectively, of the uniform flow, and

$$\psi_i(\tau) = \sum_{j=0}^{\infty} \psi_{ij}\tau^j \tag{4}$$

When K-1 terms of the series for the shock equation are used, the series for $\psi_i(\tau)$ is known through the term of degree $2K$-$2i$.

Padé fractions[11] with numerator and denominator of equal or nearly equal degrees are formed from the power series for the $\psi_i(\tau)$ and certain of their derivatives, and are used in place of the power series in the calculation of the flow. These sequences of Padé fractions are found to converge for values of τ on the body, while the power series from which they are obtained are divergent.

These approximations are used to obtain coefficients of the equation of the body, together with values of x_0/R_s and R_b/R_s. Coefficients of the expansions for the pressure, density, and velocity magnitude on the body in powers of s/R_b are then obtained, where s is the arc length measured from the nose. These expansions have the forms shown in Ref. 1, Sec. 9. Each coefficient of these expansions and of Eq. (2) can be expressed in terms of a finite number of the $\psi_i(\tau)$ and their derivatives evaluated for $\tau = \tau_0$, where τ_0 is the smallest positive root of the equation $\psi_1(\tau) = 0$. The number of coefficients of each series which can be calculated with sufficient accuracy is about half the number of terms used in Eq. (1). Finally, the partial sums are replaced by Padé fractions, and the latter are used in the flow calculations. Examples show that these rational approximations are more accurate than the partial sums near the sonic point.

Flow quantities in the shock layer are expanded in powers of $\cos\alpha$, where as defined in Ref. 1, α at a given point is the angle between the tangent to the curve τ = constant through that point and the positive x axis. Padé fractions are then formed from these expansions, and are used to calculate the flow. In particular, when 9 terms of the series for the pressure are used, we obtain a rational approximation of the form

$$p \simeq \sum_{i=0}^{4} A_i(\tau) \cos^{2i}\alpha \Bigg/ \sum_{i=0}^{4} B_i(\tau) \cos^{2i}\alpha \tag{5}$$

It follows from a property of Padé fractions that the rational approximations for u, p, ρ, q^2, and c^2 obtained in this way are exact at the shock in the inverse problem when more than three terms of the power series are used,[1] where u is the x

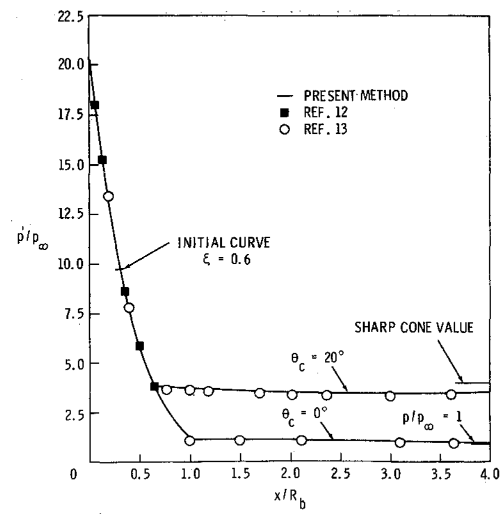

Fig. 2 Pressure on a sphere-cone and a sphere-cylinder at $M_\infty = 4$.

component of velocity, and c is the sound speed. No difficulty is found in the neighborhood of the sonic line when using these rational approximations. In order to continue the calculations by the method of characteristics, it is convenient to take a curve ξ = constant in the supersonic region as the initial curve.

In the direct problem, we use the two-parameter family of shock equations with bluntness B_s given by

$$\frac{x + x_0}{R_s} = \frac{1}{2}\left(\frac{r}{R_s}\right)^2 \frac{12B_s - (16\lambda - 3B_s^2)(r/R_s)^2}{12B_s - 16\lambda(r/R_s)^2} \tag{6}$$

A value of λ is first chosen, and B_s is found by the method of false position so that the bluntness of the calculated body matches that of the given one to a prescribed number of figures. New values of λ are then chosen until the calculated body agrees with the given one near the sonic point. A Padé fraction formed from the right side of Eq. (2) is used to obtain points on the calculated body. At least five terms of Eq. (2) are usually needed for sufficient accuracy near the sonic point.

As a guide to an initial choice of λ, we note that λ varies between -0.125 and 0.04 in the case of a sphere as M_∞ varies between 3 and ∞, and between 0.5 and 1 in the case of a spheroid of bluntness 4 when M_∞ varies between 3 and 6. An accuracy of one or two significant figures for λ is sufficient.

Special cases have been calculated on the IBM 7090 with $\gamma = 1.4$, using the present method together with Ref. 9. For a given value of K in the following examples, $K - 1$ terms of the series for the shock equation were used.

In Fig. 1, the shock, sonic line, and a typical initial curve are shown in the case of a sphere at $M_\infty = 4$, with $K = 7$, 9, 11, and $\lambda = -0.08$. Figure 2 shows the pressures on a sphere-cone of half-angle $\theta_c = 20°$ and a sphere-cylinder calculated with $K = 9$ and 11 and $\lambda = -0.08$, using the initial curve of Fig. 1 with 9 intervals of subdivision. The results are in good agreement with measurements by Xerikos and Anderson[12] on the sphere, and with calculations by Chushkin and Shulishnina.[13] The calculated pressures for $K = 7$ and $\lambda = -0.08$ agree with those shown to within the accuracy of the graph except in a small region around the initial curve.

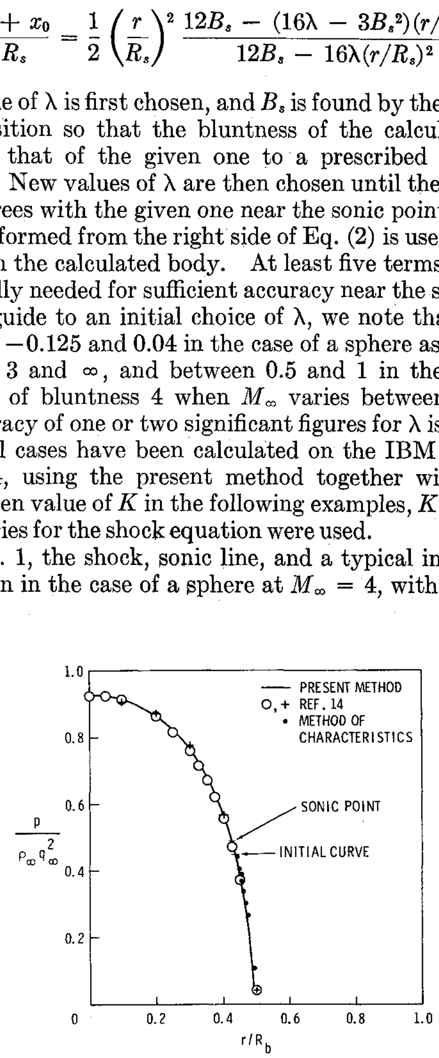

Fig. 3 Pressure on a spheroid of bluntness 4 at $M_\infty = 5.98$.

In Fig. 3, the calculated pressure on a spheroid of bluntness 4 at $M_\infty = 5.98$ is shown for $K = 9$ and 11, with $\lambda = 0.94$. Comparison is made with calculations by the method of characteristics in the supersonic region, starting from the initial curve $\xi = 0.458$ with 9 intervals of subdivision. The Mach number on the body at the initial curve is 1.0843. The two calculations differ from each other by less than 3% up to about $r/R_b = 0.46$, where the Mach number is 1.3. Good agreement with measurements by Pasiuk[14] is found.

The preceding results show that the pressure can be calculated accurately up to the sonic point and beyond with $K = 9$ when λ is known. In order to determine λ, however, it is usually necessary to take $K = 11$.

Comparison of the solution of the inverse problem for a paraboloidal shock at $M_\infty = \infty$ with calculations by Van Dyke[15] and Moran[5] shows that the calculated body is accurate to about four figures near the sonic point when $K = 9$ and 11, and to more than five figures near the nose. Similarly, when we solve the inverse problem using Moran's shock for the flow past a sphere at $M_\infty = \infty$,[5] we find that the calculated body is correct to about four figures up to the sonic point.

The solution of the direct problem for a sphere at $M_\infty = \infty$ by the present method gives $x_0/R_s = 0.099523$ and $R_b/R_s = 0.77410$ for $K = 11$ and $\lambda = 0.04$, whereas Moran[5] obtains the values $x_0/R_s = 0.099526$ and $R_b/R_s = 0.774269$.

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Measurements in the Turbulent Wake of a Sphere

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Introduction

THIS work stems from current interest in the axial decay of scalars in turbulent wakes. An axisymmetric wake was generated in a low speed, closed circuit wind tunnel using a one inch sphere suspended at the inlet of the test section with four piano wires. The test Reynolds number was 78,000.

A TSI anemometer of constant temperature type with a tungsten sensor 150 μin. in diameter was used. The intermittency circuit was similar to the one described by Corrsin and Kistler[2] and was constructed by Riddhagni. The wake was probed at 15 stations spanning 200 diam downstream of the sphere into the tunnel diffuser. Measurements made in the diffuser were later repeated in another tunnel with a longer test section. The same results were obtained.

Wake Similarity

Below the critical Reynolds Number, the boundary layer remains laminar across the front face of the sphere, separating as it enters the adverse pressure gradient on the rear face. The separated boundary layer and the flow that has been accelerated around the sphere combine to form a strong shear layer in the first diameter downstream. The mean velocity overshoots its freestream value, U_∞, within the first diameter, Fig. 1, and the turbulence intensity reaches its peak value at the end of this region.

In the next few diameters the wake becomes dynamically similar; that is, all profiles become independent of axial position when normalized with characteristic length and velocity scales, L_c and U_c. In the axisymmetric wake these scaling parameters vary as powers of x: $L_c/D \sim [(x - x_0/D)]^{1/3}$ and $U_c/U_\infty \sim [(x - x_0/D)]^{-2/3}$ where x_0 is the virtual origin of wake similarity. The decay of turbulence in the region of similarity follows a power law; for the axisymmetric wake, $(u/U_\infty) \sim (x/D)^{-2/3}$. The axial component of the turbulence intensity along the wake centerline has thus been shown as a function of x/D in Fig. 2. The straight line for $x/D > 5$ indicates similarity beyond that point. The virtual origin is found to coincide with the actual origin, in contrast to the results of Uberoi and Freymuth[3] who found $x_0 = 12D$ and similarity for $x/D > 50$ at a Reynolds number of 8,600.

Further evidence that similarity is attained within the first few diameters may be found by normalizing the centerline defect velocity, $U_0 = U_\infty - U$, and turbulence intensity, u_0, with the characteristic scale velocity as shown in Fig. 3. The lack of variation for $x/D > 5$ again shows similarity.

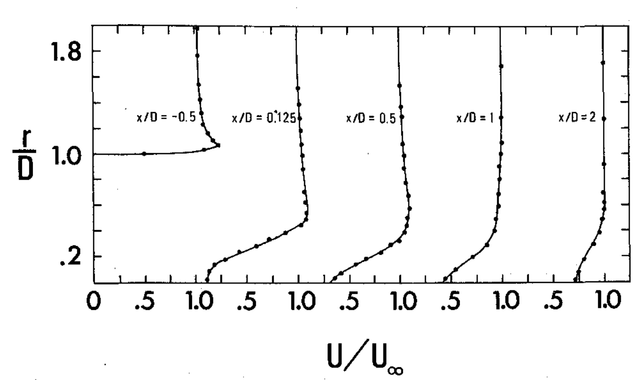

Fig. 1 Mean velocity profiles in the recirculation region.

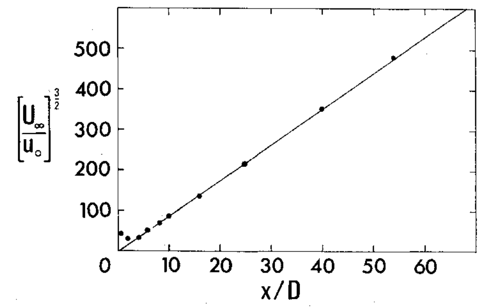

Fig. 2 The virtual origin of wake similarity shown by the axial turbulence intensity.

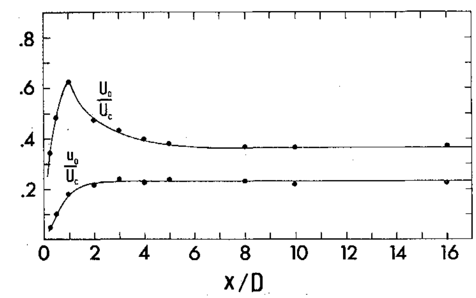

Fig. 3 The approach to similarity of the defect velocity and turbulence intensity on the wake centerline.

Wake Intermittency

Measurements of the intermittency factor made on the axis of the wake revealed that the fully turbulent core is confined to approximately the first five sphere diameters downstream. Beyond this distance the corrugation amplitude of the turbulent front has increased to the point where it is on the order of the wake radius, causing periods of laminar flow on the axis. This feature of sphere wakes was first

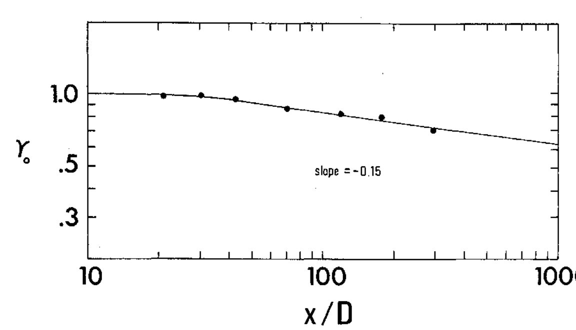

Fig. 4 The axial variation of the intermittency factor on the wake centerline.

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